Systematic modeling of the aging trend and the impact of silver economy in China

Siqi Zhou, Xuefeng Zhong, JingYing Guo

Abstract: As China rapidly transitions into a deeply aging society, the repercussions of aging on economic growth and social security systems are intensifying. This study develops a multi-module modeling framework integrating deep learning, Bayesian inference, regularization regression, and coupled differential systems to assess aging's macroeconomic and pension system impacts. Utilizing a Long Short-Term Memory neural network, we predict the proportion of those aged 65+ will surpass 19% by 2035. A happiness index for the elderly, constructed from China Health and Retirement Longitudinal Study data, highlights mental health, life satisfaction, and physical condition as key positive drivers, with notable regional variations. Elastic Net regression reveals that elderly consumption, pension participation, and service supply significantly boost gross domestic product. A coupled differential equation system indicates pension expenditure growth outpaces income, potentially creating a trillion-yuan gap by 2035, threatening system sustainability.
Keywords: Long Short-Term Memory neural networks; Bayesian statistical modeling; Elastic Net regularization regression; pension system coupling model; silver-haired economy
JEL codes: J11, H55, I31

1.Introduction

China has officially entered a stage of moderate population aging, and the resulting demographic transformation is exerting profound effects on the nation's economic and social development. According to the Seventh National Population Census, the number of people aged 65 and above has reached 191 million, accounting for 13.5% of the total population. It is projected to exceed 300 million by 2035, marking the onset of a deeply aged society. With increasing life expectancy and persistently declining fertility rates, population aging has become an irreversible trend, posing considerable challenges to pension security, healthcare resource allocation, and the broader capacity for social governance.

Against this backdrop, the "silver economy"—defined as economic activities targeted at the elderly population—has emerged as a new strategic focus and an important engine for economic growth in China. Under the dual impetus of policy guidance and market mechanisms, the silver economy has witnessed rapid expansion across a variety of sectors, including healthcare, rehabilitation, elderly tourism, and financial services. The 20th National Congress of the Communist Party of China and the Third Plenary Session of its 20th Central Committee have both emphasized the need to improve population policies, reform the elderly care service system, and promote a high-quality strategy for addressing aging. These directives provide crucial institutional support for the silver economy.

Despite growing academic interest, existing research remains limited in several key respects. First, most studies adopt a fragmented approach, focusing on isolated dimensions such as demography, economy, or policy without capturing the systemic interactions among them. Second, traditional modeling techniques struggle to reflect the nonlinear characteristics and coupled dynamics inherent in the aging process. Third, there is a lack of integration between macro-level statistical data and micro-level survey data, which constrains the explanatory power and policy relevance of the findings.

To address these limitations, this study formulates a unified framework to investigate four interrelated research questions. First, in view of the complex and nonlinear evolution of China's demographic structure, a Long Short-Term Memory (LSTM) neural network is developed to predict the trajectory of the population aged 65 and above over the next decade. Second, based on multi-dimensional individual-level data—including health status, cognitive ability, and subjective well-being—a Bayesian model is used to construct an Elderly Happiness Index and identify key influencing factors and regional disparities. Third, focusing on the economic role of the silver economy, an ElasticNet regression model is applied to quantify its contribution to GDP growth and to explore how targeted policies can drive industrial upgrading. Fourth, a coupled differential equation system is proposed to simulate the long-term evolution of the pension system by linking elderly population size, economic output, and net pension income, thereby providing a quantitative basis for institutional reform.

This research contributes to the literature in several innovative ways. First, it proposes an integrated modeling framework that connects demographic forecasting, well-being assessment, economic evaluation, and institutional response, thereby overcoming the compartmentalization seen in traditional studies. Second, it synthesizes multiple advanced methodologies—LSTM forecasting, Bayesian inference, ElasticNet regression, and dynamic system modeling—to establish a comprehensive, multi-level analytical system capable of handling complex interactions. Third, it realizes data synergy by integrating micro-level CHARLS survey data with macro-level national statistics, thus enhancing the empirical robustness and policy applicability of the analysis. Fourth, the proposed model offers strong policy simulation capabilities, particularly in forecasting pension fund gaps and simulating policy responses, thereby serving as a scientific decision-support tool for policymakers in the context of population aging.

In summary, this paper aims to provide a systematic analytical tool and quantitative decision-making support for promoting the high-quality development of the silver economy, optimizing the pension security system, and improving the well-being of the elderly population under the accelerating trend of aging in China. The study holds significant theoretical value and practical relevance.

2. Literature Review

The rapid aging of China's population has raised increasing concerns regarding its economic implications, social welfare burden, and institutional sustainability. As a result, an expanding body of research has emerged across domains such as population forecasting, elderly well-being, silver economy analysis, and pension system reform. This section reviews the most relevant literature, focusing on four major strands: aging trend prediction, elderly life quality modeling, economic impacts of population aging, and simulation of pension system dynamics.

2.1 Aging Trend Forecasting Models

Accurate forecasting of the elderly population is crucial for informing long-term planning in health care, pensions, and labor markets. Early research predominantly relied on classical statistical models such as linear extrapolation, ARIMA, and exponential smoothing, which are easy to implement but often struggle with nonlinear trends and structural shifts. Cohort-component models, including the widely adopted Lee–Carter model (Lee & Carter, 1992), offer demographic rigor by incorporating birth, death, and migration rates but face limitations in high-frequency policy simulation.

More recent advances in machine learning have introduced deep neural networks—particularly Long Short-Term Memory (LSTM) networks—as a powerful tool for nonlinear time-series modeling. LSTM models are capable of learning long-range temporal dependencies, making them particularly suited for capturing the evolving and nonlinear nature of China’s demographic transition. Several empirical studies have demonstrated the superior performance of LSTM over traditional methods in forecasting age-structured population changes (Zhang & Li, 2023; Xu & Huang, 2024). However, few have integrated such forecasts into economic modeling frameworks or policy simulations.

Our study builds on this literature by applying LSTM to predict the proportion of the population aged 65 and above and then linking this trajectory to downstream economic and institutional variables, such as GDP growth and pension expenditure.

2.2 Elderly Life Quality and Bayesian Well-being Models

Subjective well-being among older adults has gained increasing attention as a measure of successful aging. The OECD’s guidelines on measuring subjective well-being emphasize multidimensional indicators, including life satisfaction, psychological resilience, physical functioning, and perceived health. In the Chinese context, studies utilizing the China Health and Retirement Longitudinal Study (CHARLS) have analyzed the effects of health, family support, and income on elderly happiness, often employing linear regression or structural equation models (Liu & Sun, 2023; Li & Zhao, 2022).

Despite these advances, traditional estimation methods such as OLS are sensitive to multicollinearity and may fail to capture parameter uncertainty in high-dimensional settings. Bayesian regression, by contrast, offers a probabilistic approach that incorporates prior knowledge and yields credible intervals for parameter estimates. This is particularly useful in social science applications involving noisy, correlated, and heterogeneous data.

Our work adopts a Bayesian framework to construct a composite Elderly Life Quality Index (ELQI), integrating psychological health, life satisfaction, and pain status. This allows for posterior inference on key predictors while accommodating uncertainty, enhancing the interpretability and robustness of policy implications.

2.3 Economic Impact of Aging and the Silver Economy

The economic consequences of population aging are complex and multi-faceted. While a shrinking labor force may slow economic growth, the rise of the silver economy—comprising sectors such as eldercare, health services, financial products, and age-friendly housing—presents new opportunities for demand-side expansion. Empirical studies have produced mixed findings on the net effect of aging on GDP growth, with results varying by context, modeling strategy, and data availability (Bloom et al., 2010; Zhou & Zhao, 2024).

Standard econometric models such as linear regression and vector autoregression have been widely used to estimate these effects. However, these approaches are often constrained by multicollinearity among variables such as health expenditure, pension coverage, and elderly consumption. Regularization techniques like the ElasticNet regression, which combines L1 and L2 penalties, offer a flexible solution by enabling both variable selection and shrinkage (Zou & Hastie, 2005).

In our study, we apply the ElasticNet model to a set of macro- and micro-level indicators derived from national statistics and CHARLS data, identifying the most influential components of the silver economy and quantifying their impact on GDP. This contributes to a more precise understanding of how aging-related variables interact with macroeconomic outcomes.

2.4 Modeling Pension System Dynamics with Differential Equations

The sustainability of pension systems under demographic stress is a central topic in both academic and policy debates. Previous studies have applied static actuarial models, computable general equilibrium (CGE) models, and micro-simulation frameworks to project pension fund balances. While these models offer valuable insights, they often assume linear evolution and lack endogenous feedback mechanisms between demographic, economic, and fiscal variables.

Recent research has highlighted the potential of differential equation systems to simulate dynamic interactions in pension systems (Hu & Zheng, 2024; Wang & Zhao, 2024). By treating pension revenue, expenditure, and elderly population as interdependent state variables, such models can capture the non-linear and path-dependent nature of institutional change.

In this context, we construct a coupled differential equation model that links the growth rate of the elderly population, average consumption, GDP, and pension fund inflows and outflows. Parameters are estimated via numerical optimization to minimize prediction error over historical data. This approach allows for scenario analysis under different policy regimes—such as delayed retirement or contribution rate adjustment—providing a flexible and realistic framework for long-term pension sustainability evaluation.

3. Data and Variable Description

To construct a comprehensive analytical framework encompassing aging trend forecasting, subjective well-being modeling, silver economy impact estimation, and pension system simulation, this study integrates multiple data sources, including national macroeconomic statistics (1990–2022), the China Health and Retirement Longitudinal Study (CHARLS 2020), and policy-based model specifications. The key variables and their definitions for each modeling module are described as follows.

3.1 Variables for Aging Trend Forecasting

This section applies a Long Short-Term Memory (LSTM) neural network to forecast the long-term evolution of the population aged 65 and above as a share of the total population. The key variables include:

·Aging Rate: The proportion of the population aged 65 and above, serving as the dependent variable.
·Time (Year): Calendar year as the independent variable forming the time window input for the LSTM model.
The data are obtained from the China Statistical Yearbook and the United Nations World Population Prospects. A univariate time series model is constructed to capture the nonlinear dynamics and long-term patterns of population aging.

3.2 Variables for Happiness Index Construction
To quantify the subjective well-being of the elderly, this study constructs a proxy variable — Happiness Proxy — based on micro-level data from CHARLS 2020. This composite indicator captures multiple dimensions of well-being and health status, characterized by:
·Mental Health (Positive): Inverted CES-D score to indicate positive emotional status;
·Self-Rated Health: 1–5 ordinal scale for perceived health;
·Physical Pain (Positive): Pain levels recoded as comfort indicators;
·ADL/IADL: Activities of Daily Living and Instrumental Activities of Daily Living, used for robustness checks.
This module employs a Bayesian regression model to analyze the relationship between happiness and health status, while identifying key determinants influencing subjective well-being.

3.3 Variables for Silver Economy Impact Estimation
This section utilizes macroeconomic panel data and applies ElasticNet regression to estimate the impact of silver economy-related variables on GDP growth. The dependent variable is:
·Gross Domestic Product (GDP): Measured in current prices (trillion RMB).
The explanatory variables are listed in Table 1 and classified into four categories:

Table 1.Variables related to silver economy

Category	Example Variables	Symbols
Demographic Structure	Population aged 65 and above, Elderly share of total population, Elderly dependency ratio	x_1,x_2,x_3
Consumption & Expenditure	Per capita elderly consumption, healthcare expenditure, number of elderly care beds	x_4,x_5,x_9
Social Security	Pension participants, fiscal expenditure on elderly care, basic medical insurance	x_6,x_7,x_8
Health & Longevity	Life expectancy, number of elderly care institutions	x_{10},x_{11}
Industrial & Economic Base	Share of tertiary industry, fixed asset investment, total imports	x_{12},x_{13},x_{15}
Household & Aggregate Economy	Per capita household consumption, total population	x_{14},x_{16}

To address multicollinearity, all variables are standardized and log-transformed. ElasticNet integrates both L1 and L2 regularization to achieve efficient variable selection and robust multifactor estimation.

3.4 Variables for Pension System Simulation

This section builds a system of ordinary differential equations (ODEs) to simulate the dynamic interaction among population aging, public finance, and pension sustainability. The core variables include:

·Aging Rate *A(t)*: Proportion of population aged 65+,representing demographic transition;

·Elderly Consumption *C(t)*: Per capita consumption by the elderly;

·Medical Expenditure *M(t)*: Reflects healthcare fiscal pressure;

·Pension Net Outflow *P(t)*: Defined as pension expenditure minus revenue;

·Pension Revenue P_{in}: Exogenous input variable modeled as a linear function $P_{in}(t) = \theta_0 + \theta_1 t$;

These variables form a four-equation coupled system to reflect the behavioral and structural responses of the pension system to population aging. The model simulates the projected fiscal gap over the next 50 years, offering quantitative insights for policy reform.

4. Modeling Methods and Analytical Framework

4.1 LSTM Modeling and Aging Trend Forecasting

As China's demographic structure undergoes significant transformation, the rising proportion of the population aged 65 and above poses major implications for pension systems, public healthcare, and economic sustainability. Accurately forecasting the trajectory of the elderly population share is essential for informed policy design and strategic planning. Traditional time series models, such as ARIMA or autoregressive frameworks, provide limited capabilities in capturing structural shifts, nonlinear patterns, and long-range dependencies. To address these limitations, this study introduces a Long Short-Term Memory (LSTM) neural network to construct a memory-enhanced nonlinear forecasting model capable of learning complex population dynamics.

4.1.1 Data Source and Preprocessing

The population data used for model training are obtained from the China Statistical Yearbook, Population and Employment Statistics Yearbook, and the United Nations World Population Prospects, covering the years 1990 to 2020. The series represents the annual share of people aged 65 and above in China's total population. The Augmented Dickey-Fuller (ADF) test reveals that the raw series is non-stationary (p-value = 0.9994); after first-order differencing, the p-value drops to 0.0109, confirming stationarity for subsequent modeling.

To enhance training efficiency and convergence, the series is normalized to a [0, 1] range. A sliding window approach is adopted, with a lag order of 2, forming input sequences $[x(t-2), x(t-1)]$ and corresponding target output $x(t)$ for supervised learning.

4.1.2 Model Architecture and Parameter Settings

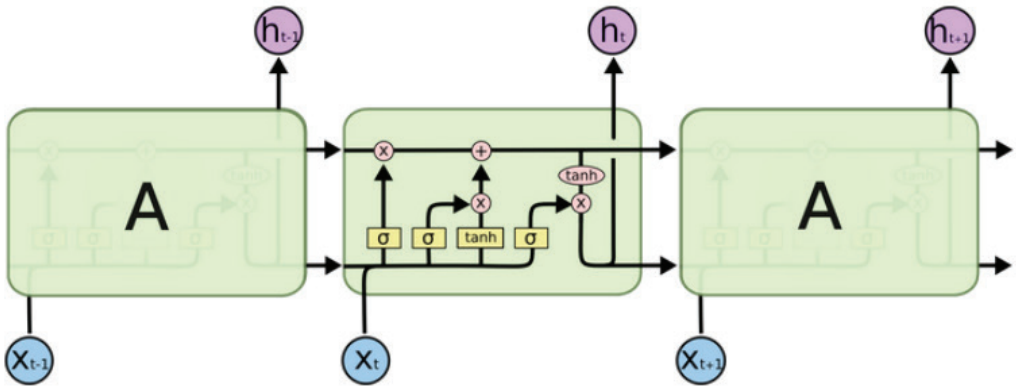
As shown in Figure 1, the LSTM prediction model constructed in this paper contains the following structure:

·Input Layer: Shape of (2,1), corresponding to two lagged time steps.

·LSTM Layer: 50 memory units, with tanh activation.

·Dropout Layer: Dropout rate set at 0.2 to prevent overfitting.
·Dense Layer: Outputs a single real-valued prediction.
·Loss Function: Mean Squared Error (MSE).
·Optimizer: Adam optimizer.
·Training Configuration: 100 epochs and a batch size of 16.
The model is implemented in the Keras framework. The training process exhibits a steady reduction in loss, indicating stable convergence.

Figure 1.Diagram of LSTM neural network structure



4.1.3 LSTM Mechanism and Mathematical Formulation

The LSTM network overcomes the vanishing gradient problem encountered in traditional RNNs by introducing gate mechanisms that control memory retention and update across time steps. The key computations for each time step ttt are as follows:

·Forget gate:

$$f_t = \sigma\left(W_f \cdot [h_{t-1}, x_t] + b_f\right) \tag{1}$$

·Input gate and candidate state:

$$i_t = \sigma(W_i \cdot [h_{t-1}, x_t] + b_i), \widetilde{C}_t = \tanh\,(W_c \cdot [h_{t-1}, x_t] + b_C) \tag{2}$$

·State update and output gate:

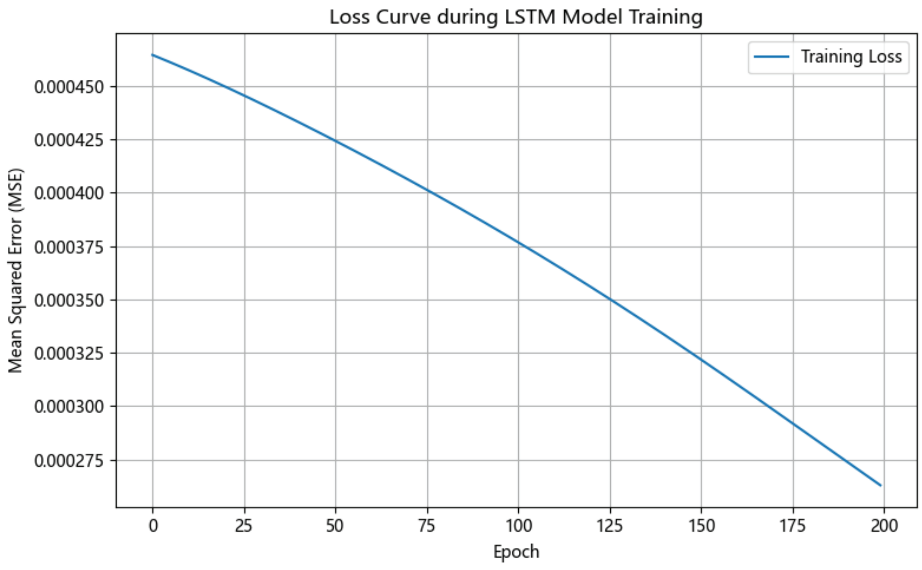
$$C_t = f_t \cdot C_{t-1} + i_t \cdot \widetilde{C}_t, o_t = \sigma(W_o \cdot [h_{t-1}, x_t] + b_o), h_t = o_t \cdot \tanh\,(C_t) \tag{3}$$

Here, x_t denotes the current input, h_{t-1} the previous hidden state, $\sigma(\cdot)$ the sigmoid function, and C_t the cell state. Parameters W and b represent the model weights and biases.These mechanisms allow the model to capture both short-term variations and long-term dependencies in the aging population sequence.

4.1.4 Model Fitting Performance and Forecast Results

Figure 2 illustrates the model's loss convergence curve during training, indicating high stability and learning efficiency.

Figure 2.Training loss convergence curve



The model achieves strong out-of-sample performance, with evaluation metrics summarized as follows:

Table 2.Evaluation indexes of the LSTM model

Metric	MSE	RMSE	MAE	*R*2
LSTM Forecast	0.071192	0.0106	0.0085	0.987

Figure 3 displays the scatter plot of predicted versus actual values, with points clustering near the 45-degree line, indicating strong goodness-of-fit.

Figure 3. Predicted vs. True Values plot

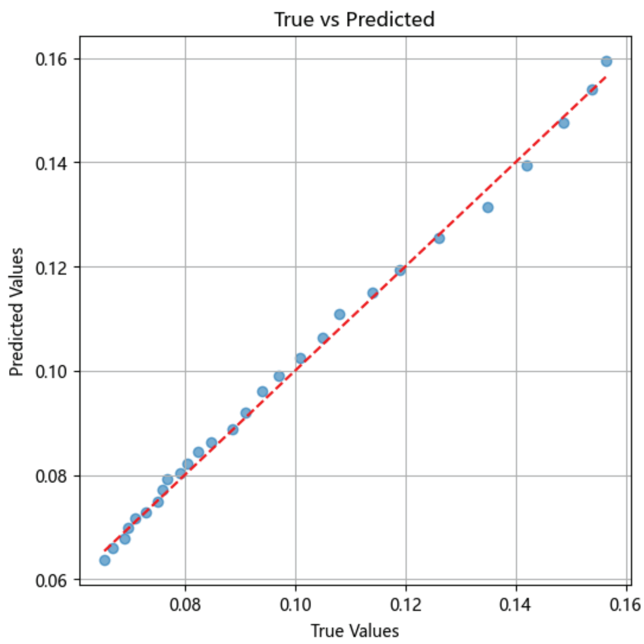
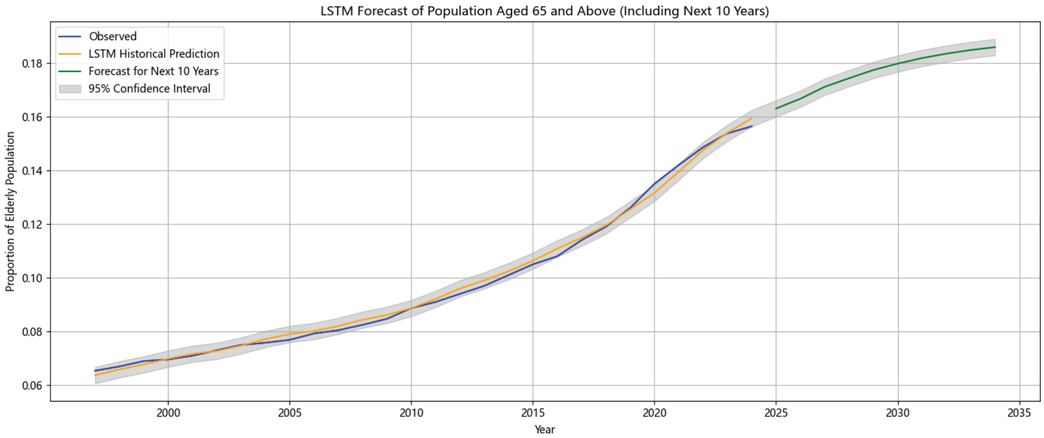


Figure 4 presents the LSTM model's forecast of the proportion of elderly population from 2021 to 2035, projecting a steady increase from 14.9% to approximately 22.6%, reflecting an accelerated aging trajectory.

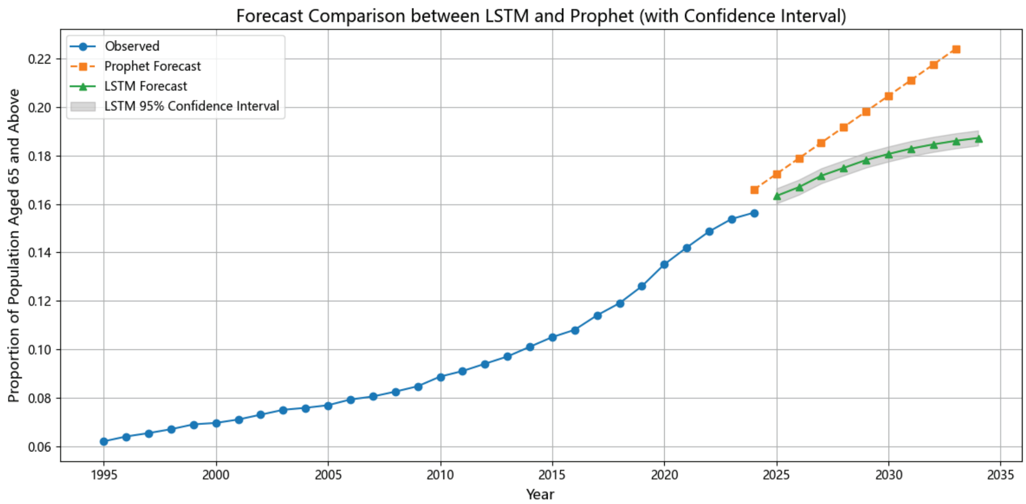
Figure 4. Projected Ageing Population 2021–2035



4.1.5 Comparative Analysis and Robustness Check

To evaluate robustness and benchmark the LSTM model, we introduce the Prophet model developed by Facebook as a comparator. Prophet is a decomposition-based forecasting tool suitable for sequences with seasonality and trend. As shown in Figure 5, the LSTM model outperforms Prophet in capturing recent structural shifts and trend inflection points, especially in the post-2015 period. Therefore, LSTM is adopted as the primary forecasting model in this study due to its superior adaptability and predictive accuracy.

Figure 5. LSTM vs Prophet forecast comparison



4.2 Construction of the Happiness Index and Bayesian Analysis

4.2.1 Variable Construction and Happiness Index System Design

In assessing the overall well-being of the elderly population, single-dimensional indicators often fail to comprehensively reflect their physical health, mental state, and subjective life satisfaction. To construct a representative happiness proxy, this study integrates four critical dimensions—life satisfaction, mental health, self-rated health, and physical pain—formulated as follows:

$$Happiness\ Proxy = \frac{Life\ Satisfaction + Mental\ Health + Self - rated\ Health + Physical\ Pain}{4} \qquad (4)$$

This indicator possesses the following characteristics:
·Multidimensional coverage: It reflects both subjective health evaluation (self-rated health) and objective physical perception (pain levels).
·Psychological integration: It incorporates both mental state and overall life assessment.
·Directional consistency: All components are positively oriented; a higher score indicates a stronger sense of happiness.
On this basis, the variables covering health, functional status and emotional feelings in the CHARLS data were selected to construct a complete well-being index system. The description and coding of the main variables are shown in Table 3, which involve ADL, IADL functional limitations, self-health perception and pain frequency, etc., to ensure a broad description of the level of well-being.

Table 3.Explanation of variable coefficients of Bayesian regression model

Regression coefficients	Corresponding variables and interpretations
β_0	The model's baseline happiness index level (expected value when all independent variables are zero)
β_1	Mental Health_Positive (mental health status score, higher indicates better mental state)
β_2	Life satisfaction (subjective life satisfaction score)
β_3	Body Pain_Positive (inverse score of body pain level, higher indicates less pain)
β_4	Hopeful for the future (rating for optimism about the future)
β_5	ADL_ positive (score of activities of daily living, higher indicates greater self-care ability)
β_6	Sleep duration (average number of hours of sleep per day)
β_7	IADL_ positive (instrumental ability score for activities of daily living, e.g. shopping, cooking, etc.)
β_8	Cognitive ability (0~21 points, the higher the score, the better the cognitive function)
β_9	Episodic memory (0~10 points, the higher the score, the stronger the memory ability)

4.2.2 Modeling Approach and Bayesian Estimation Framework

To quantify the marginal effects of each factor on elderly happiness, we apply the following linear Bayesian regression model:

$$y_i = \beta_0 + \sum_{j=1}^{p} \beta_j x_{ij} + \varepsilon_i\,, \varepsilon_i \sim \mathrm{N}(0, \sigma^2) \qquad (5)$$

Here, y_i denotes the happiness proxy for individual i, x_{ij} is the j-th explanatory variable, β_j is the coefficient to estimated, and ε_i is the error term.Priors are specified as：

$$\beta_j \sim N(0,1), \sigma^2 \sim Inverse - Gamma(2,2) \qquad (6)$$

The posterior distribution is obtained via Bayes' theorem:

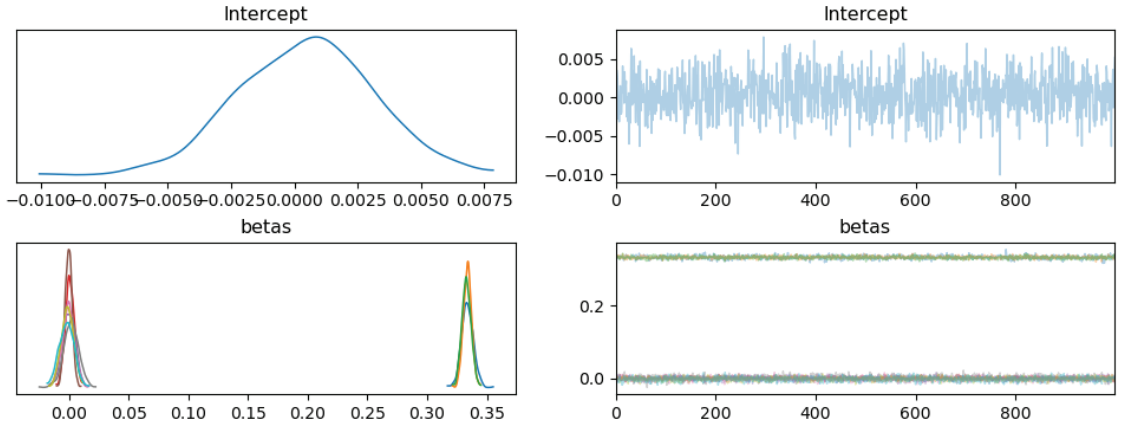
$$p(\beta_0, \beta, \sigma | y, X) \propto p(y | X, \beta_0, \beta, \sigma) \cdot p(\beta_0) \cdot p(\beta) \cdot p(\sigma) \quad (7)$$

Given the potential multicollinearity and complexity of the variables, we adopt Automatic Differentiation Variational Inference (ADVI) instead of traditional MCMC to enhance computational efficiency and model stability.

4.2.3 Convergence Diagnosis and Posterior Analysis

Model convergence is assessed using posterior density plots, sampling trajectories, and Rhat statistics. All Rhat values are close to 1.0, indicating convergence and credible estimation. As shown in Figure 6, posterior distributions are unimodal and symmetric, while sampling trajectories are stable without divergence.

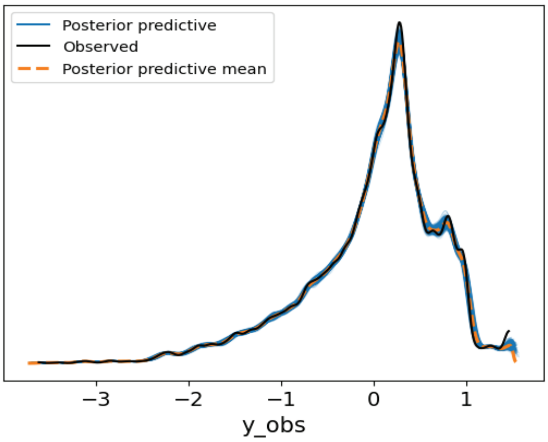
Figure 6. Posterior Density and Sampling Trajectory of Bayesian Parameters, verifying model estimation effectiveness.



4.2.4 Posterior Predictive Check

To further validate the model's predictive capacity, a posterior predictive check was conducted against actual observed data. Figure 7 shows that the posterior predictions align well with the observed values, with residuals approximately normally distributed. This demonstrates strong out-of-sample generalizability and robust inferential capability.

Figure 7. Posterior Predictive Distribution vs. Observed Values, confirming the model's fit and predictive validity.



4.2.5 Result Interpretation and Policy Implications

Compared to traditional MCMC methods, ADVI significantly enhances computational efficiency while maintaining inference accuracy. The posterior distribution provides an estimate of parameter uncertainty, which helps us assess which factors significantly influence the happiness of the elderly. The posterior distribution statistics for each regression coefficient are presented in Table 4:

Table 4.Summary of posterior distribution results of parameters

	mean	sd	hdi_2.5%	hdi_97.5%	mcse_mean	mcse_sd	ess_bulk	ess_tail
β_0	0.000	0.003	-0.005	0.005	0.0	0.0	1022.0	983.0
β_1		0.005	0.324	0.342	0.0	0.0	1028.0	1024.0
β_2		0.004	0.326	0.340	0.0	0.0	923.0	874.0
β_3		0.004	0.325	0.340	0.0	0.0	951.0	908.0
β_4	-0.001	0.004	-0.008	0.007	0.0	0.0	1018.0	783.0
β_5	-0.002	0.005	-0.011	0.018	0.0	0.0	1068.0	974.0
β_6	0.001	0.003	-0.005	0.006	0.0	0.0	1061.0	908.0
β_7	-0.002	0.005	-0.012	0.007	0.0	0.0	1063.0	981.0
β_8	0.005	0.007	-0.007	0.018	0.0	0.0	1070.0	1018.0
β_9	-0.004	0.005	-0.014	0.006	0.0	0.0	1004.0	909.0
β_{10}	-0.005	0.006	-0.016	0.008	0.0	0.0	981.0	1037.0

pour：** indicates that 95%HDI does not include 0;

As shown in Table 4,the resulting linear expression for the happiness index is:

$$Happiness = 0.334X_1 + 0.334X_2 + 0.333X_3 \tag{8}$$

Where:

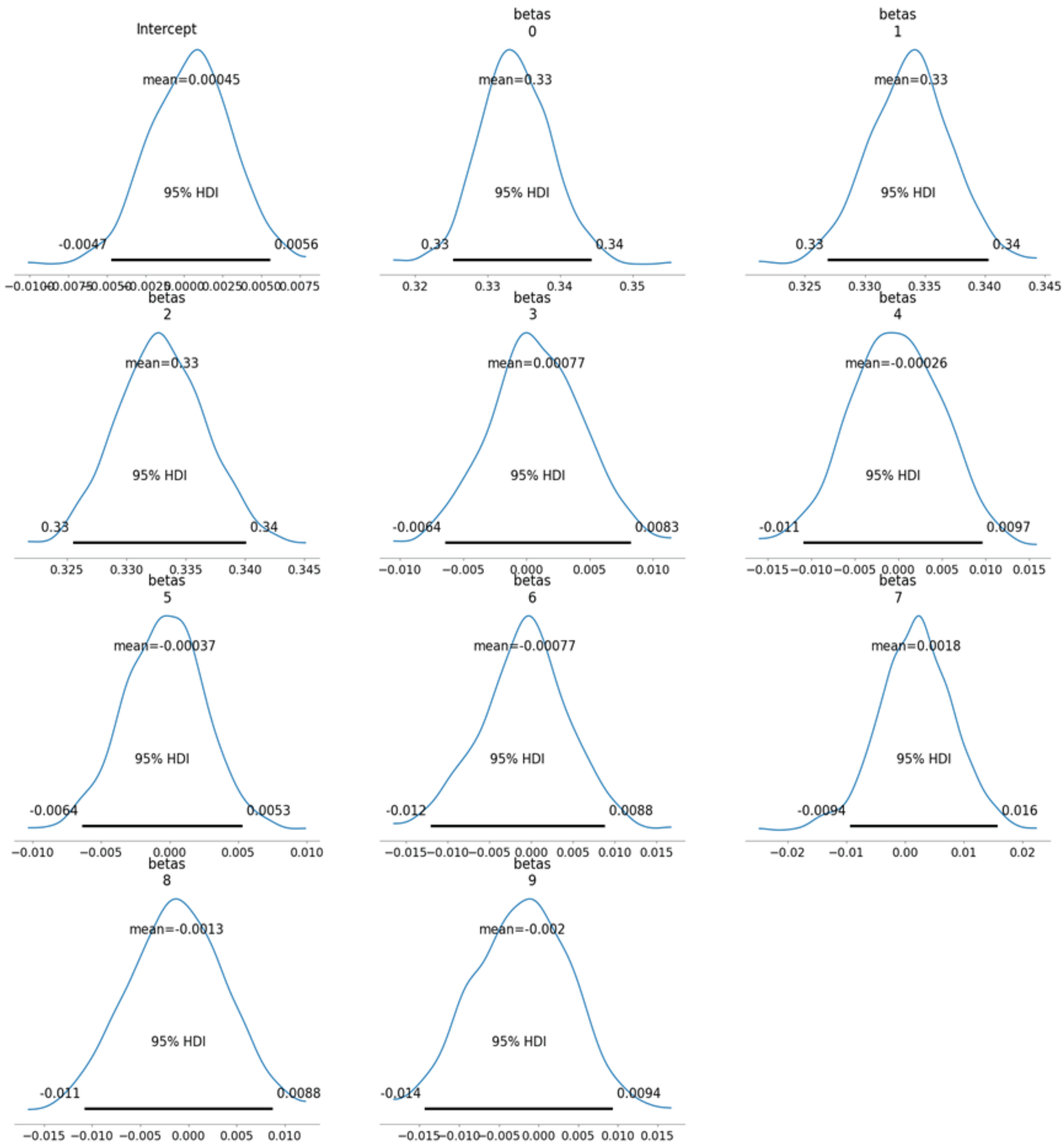
·X_1: Mental health(positive indicator)

·X_2: Life satisfaction(positive indicator)

·X_3: Physical oain relief(positive indicator)

As shown in Figure 8,The 95% Highest Density Interval (HDI) for all three variables excludes zero, indicating statistically significant positive effects on happiness ($p<0.05$). Moreover, the coefficients are highly similar, underscoring the equal contribution of these psychological and physical dimensions.Other variables (e.g., ADL, IADL, social participation) failed to pass the significance test, with HDI intervals covering zero. These factors may exhibit instability or weak effects, suggesting they should be removed or recoded in future model iterations.

Figure 8.Posterior distribution of Bayesian model parameters



This finding underscores the central role of mental wellness, perceived satisfaction, and physical comfort in shaping elderly well-being. The results align with prevailing theories in health economics and well-being literature and offer a concrete basis for targeted aging policy.

4.2.6 Spatial Disparities and Regional Determinants of Happiness Index

To further explore the spatial heterogeneity in elderly happiness across regions in China, this section integrates the estimation results of the Bayesian regression model with Kernel Density Estimation (KDE), radar plots, and boxplots to uncover geographic patterns and their underlying determinants.

As shown in Figure 9, the KDE curves indicate that developed eastern and southern coastal provinces exhibit more concentrated and higher average levels of elderly happiness. In contrast, western provinces such as Tibet, Qinghai, and Gansu display lower happiness levels with greater variability. These spatial patterns are further illustrated in Figure 10, where the radar chart highlights considerable regional disparities across key happiness dimensions—most notably in mental health and physical well-being—underscoring the uneven distribution of psycho-social support services.

Figure 9.Kernel Density Estimation of Happiness Index by Province

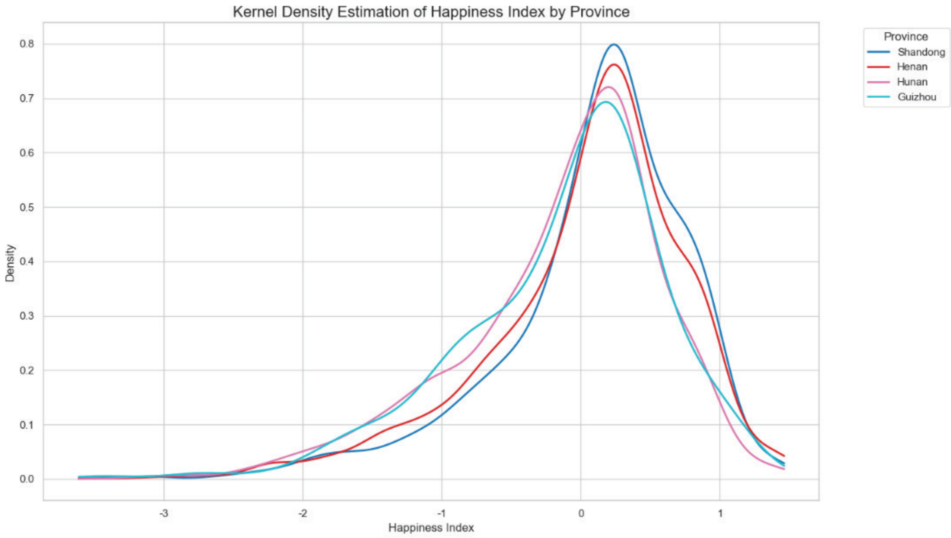


Figure 10.Radar Chart of Key Indicators Affecting Happiness Index by Province

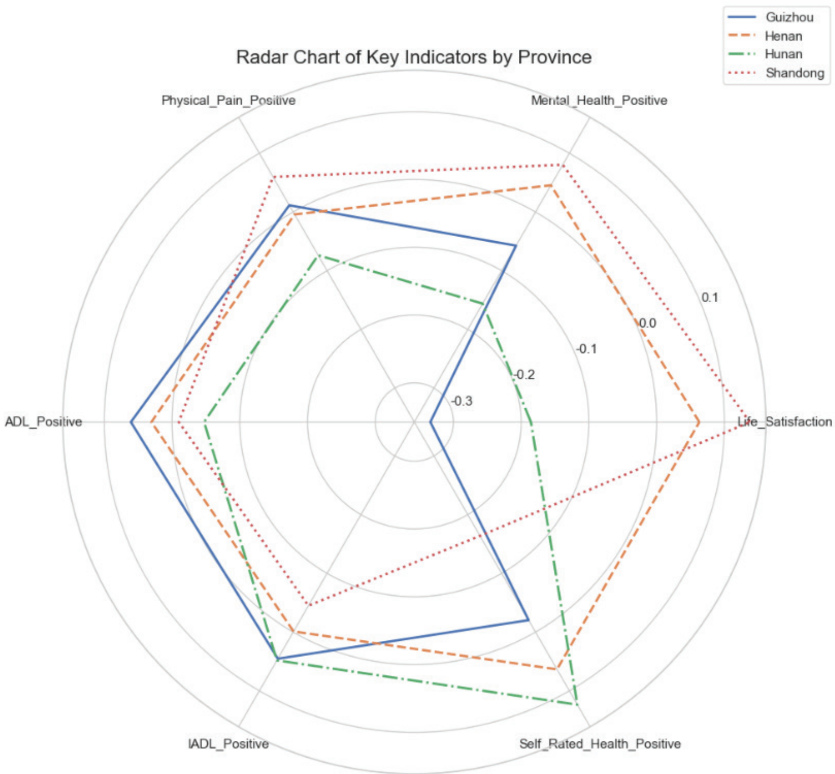
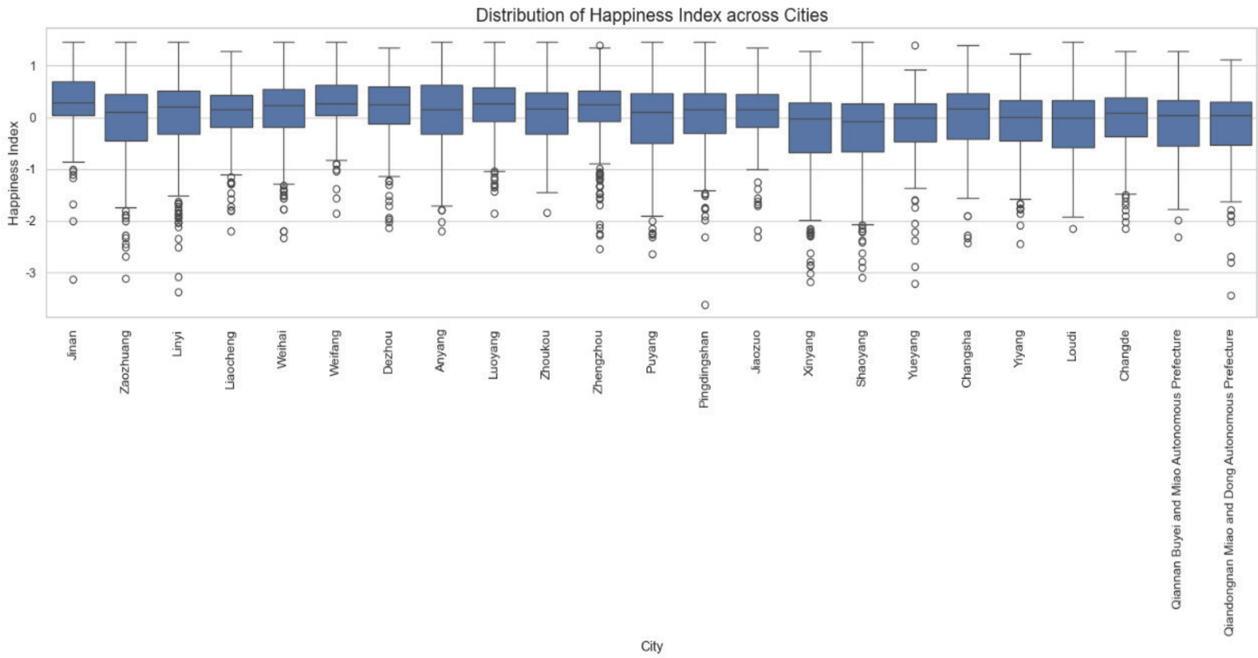


Figure 11 presents boxplots comparing the distribution of happiness indices across different urban agglomerations. The findings reveal that elderly individuals in metropolitan clusters such as Beijing–Tianjin–Hebei, the Yangtze River Delta, and the Pearl River Delta enjoy higher levels of subjective well-being than their counterparts in western and inland regions. This suggests that regional disparities in economic development and service provision significantly shape elderly happiness outcomes.

Figure 11.Box Plot of Happiness Index Distribution Across Different Cities



In sum, the spatial variation in happiness reflects structural differences in economic foundation, healthcare accessibility, social inclusion, and mental health services across regions. These findings underscore the need for differentiated regional policy strategies that prioritize equitable infrastructure investment, community engagement, and age-friendly environments to promote inclusive and balanced development.

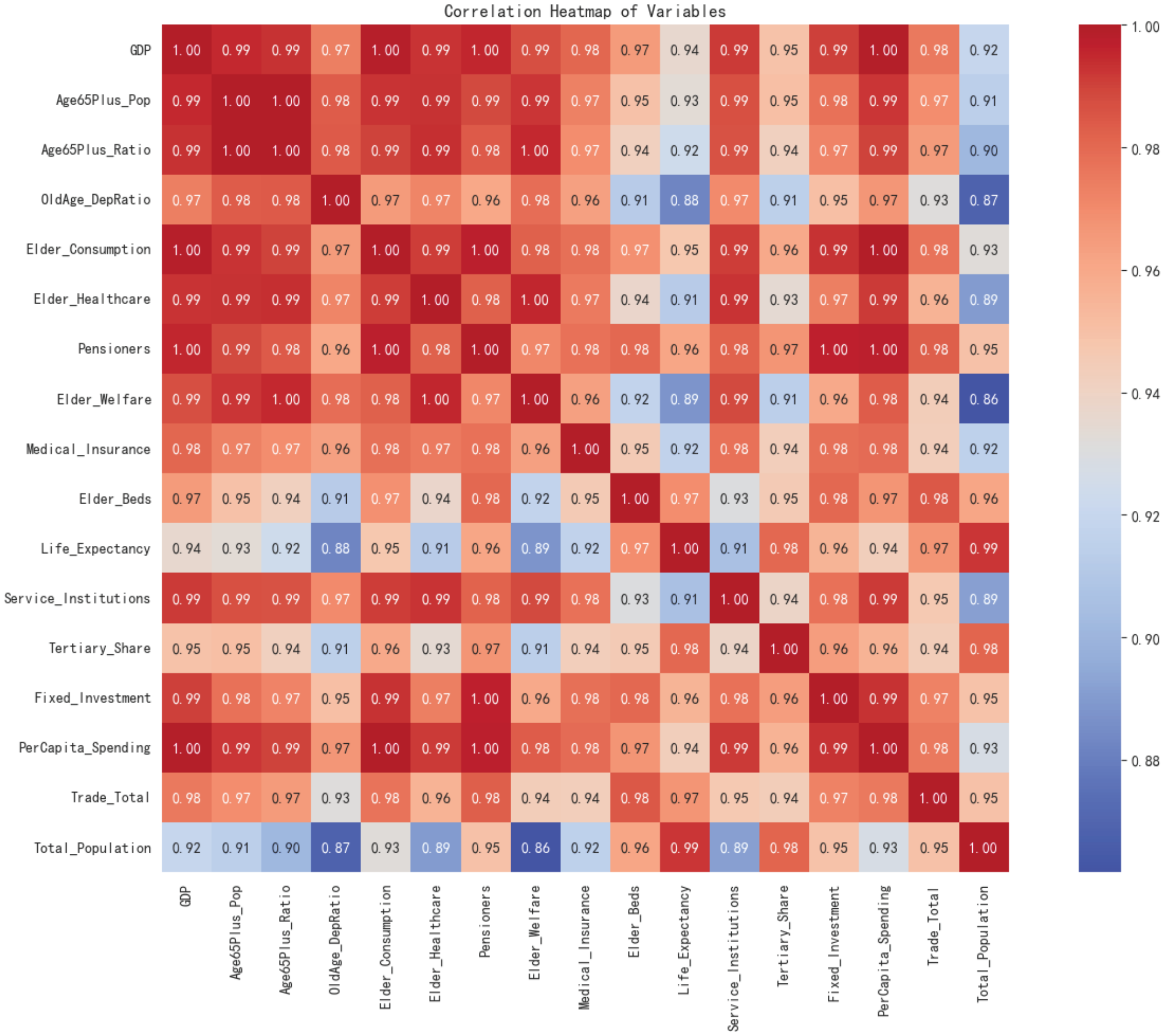
4.3 Modeling the Silver Economy Using ElasticNet Regression

4.3.1 Motivation and Variable Construction

As China moves into a stage of deep population aging, the silver economy has become a crucial driver of economic transformation. It encompasses not only traditional elderly care and medical services but also emerging sectors such as elder-targeted consumption, financial services, and assistive technology. Based on literature review and policy analysis, this study develops an 18-variable indicator system reflecting four dimensions: demographic structure, health security, economic participation, and institutional provision. Variable definitions are listed in Table 1.

To gain an initial understanding of inter-variable relationships, we conducted a correlation analysis, as visualized in Figure 12. The results show significant positive and negative correlations among key indicators. For instance, pension expenditures are strongly associated with healthcare inputs, and the old-age dependency ratio correlates negatively with fiscal health. These findings suggest a high degree of multicollinearity, warranting the use of regularization techniques to enhance modeling robustness and explanatory power.

Figure 12.Variable correlation heat map



4.3.2 Methodology and Theoretical Foundations

To handle the multicollinearity and potential overfitting caused by high-dimensional variables, we apply the ElasticNet regression model, which combines the strengths of both Lasso and Ridge regression. The loss function is defined as follows:

$$\min_{\beta}\left(\frac{1}{2n}\sum_{i=1}^{n}(y_i - X_i\beta)^2 + \lambda_1\|\beta\|_1 + \lambda_2\|\beta\|_2^2\right) \tag{9}$$

Here, $\|\beta\|_1$ denotes the Lasso penalty,which performs variable selection,while $\|\beta\|_2^2$ represents the Ridge penalty, which addresses multicollinearity.The regularization parameters λ_1 and λ_2 are determind via cross-validation.

For methodological comparison, we implemented and visualized Lasso (Figure 13) and Ridge (Figure 14) regressions. Lasso provides variable sparsity but may discard correlated features, whereas Ridge maintains variable stability at the cost of less interpretability.

Figure 13.Lasso coefficient path

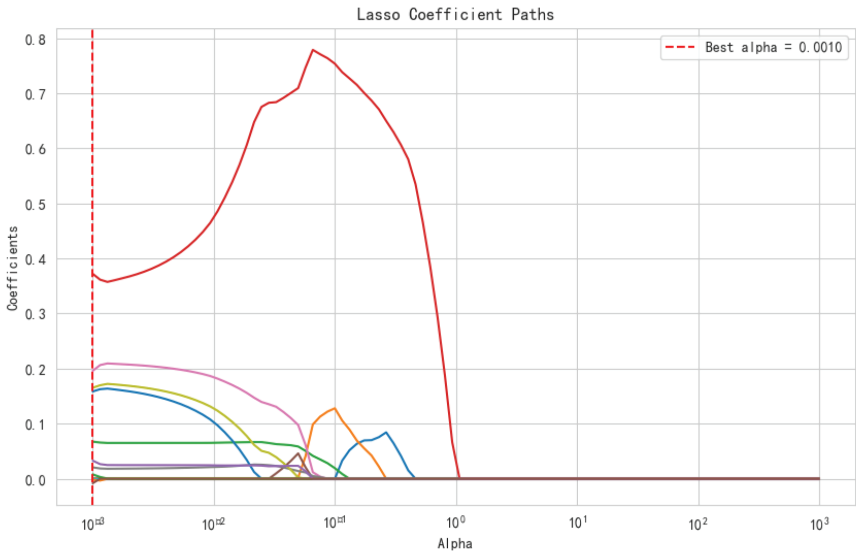
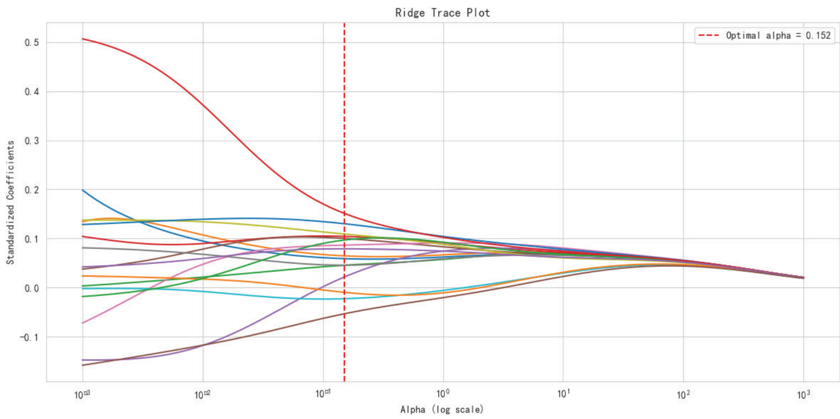
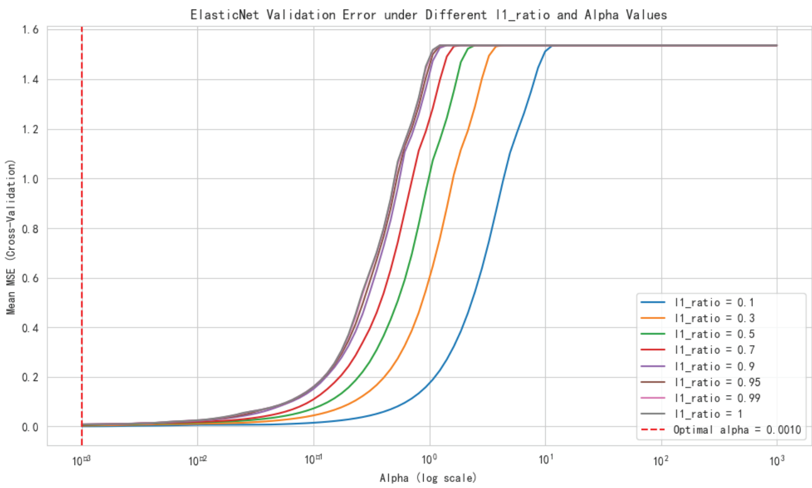


Figure 14.Lasso coefficient path



As shown in Figure 15, ElasticNet strikes a balance between the two and is therefore adopted for final modeling.

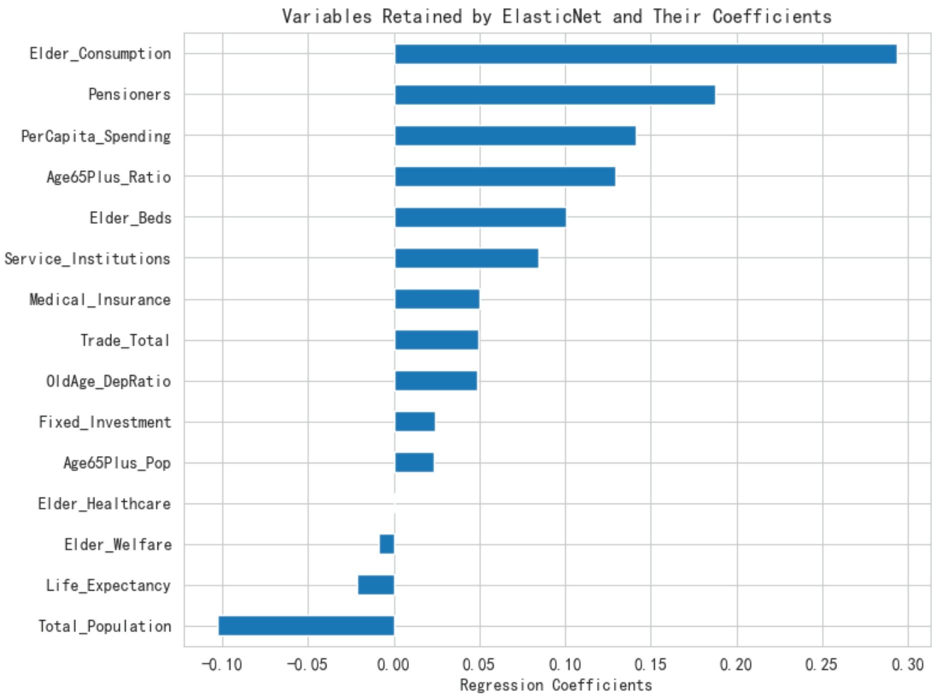
Figure 15.Verification errors of ElasticNet at different l1_ratio and alpha



4.3.3 Estimation Results and Economic Implications

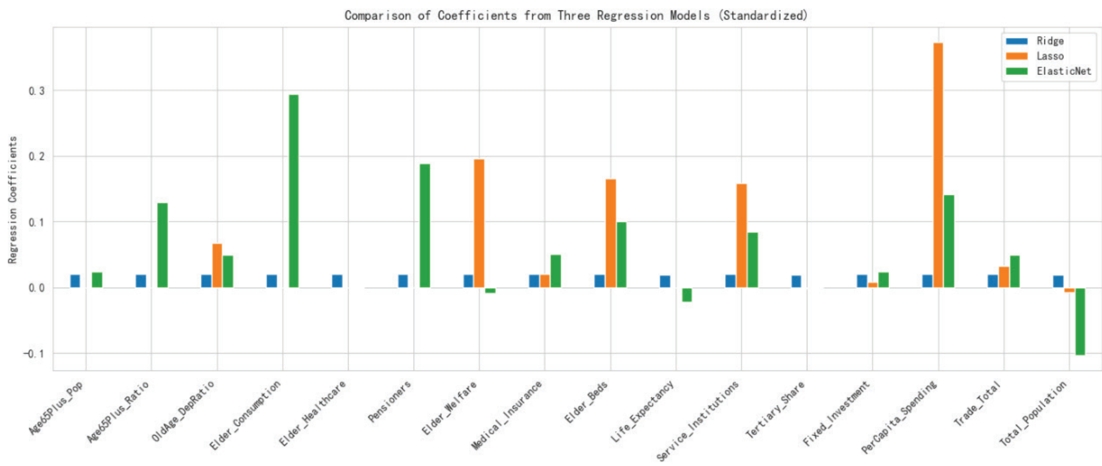
The ElasticNet model yielded a stable set of estimates following optimal parameter tuning. Key results are visualized in Figure 16, indicating that variables such as per capita pension, elderly consumption, and social security coverage positively influence GDP, whereas high old-age dependency and aging ratios exert negative effects. This duality underscores the developmental potential and structural burden embedded in the silver economy.The top five predictors-primarily from the domains of elderly welfare and consumption-highlight the dual-engine nature of the silver economy.

Figure 16.ElasticNet model retention variables and coefficients



Comparative results of variable selection across models are shown in Figure 17, confirming ElasticNet's effectiveness in balancing model simplicity and explanatory depth.

Figure 17.Comparison of coefficients of the three regression models



These findings imply that policies should enhance health service provision, expand supply of elder-targeted goods, and reinforce the pension system to enable greater consumption capacity and economic participation among the elderly.

4.3.4 Forecasting and Policy Implications

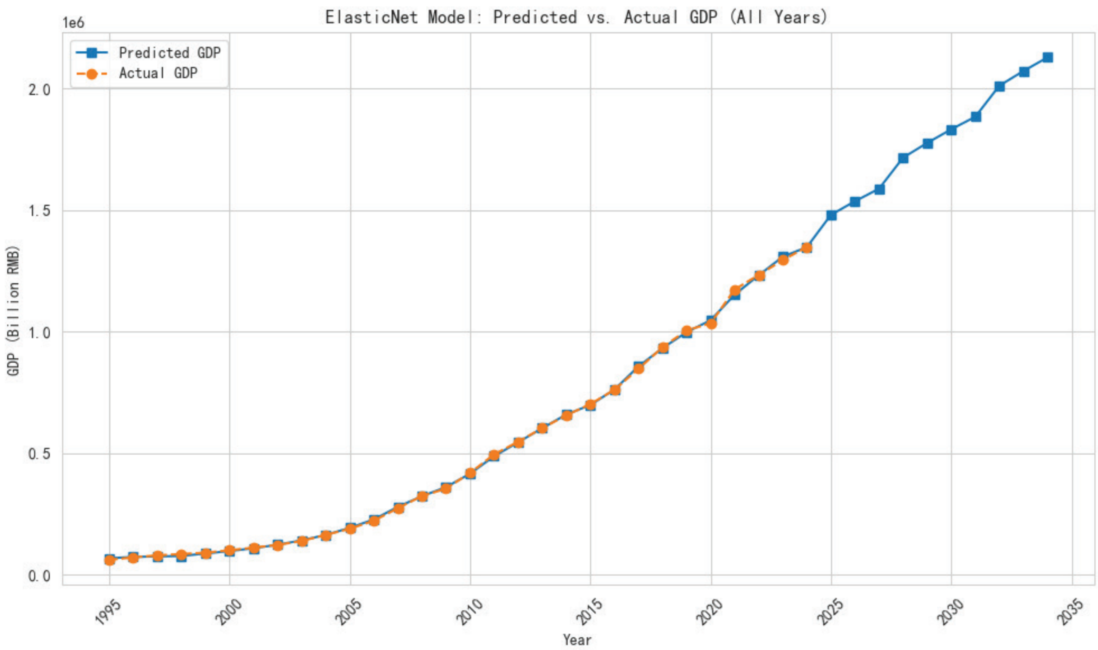
In this study, we aim to construct a GDP prediction model based on variables related to an aging society. When forecasting GDP from 2025 to 2034, as shown in Table 5, the Prophet model was used to predict key independent variables, such as elderly consumption expenditure, healthcare coverage, and dependency ratio, one by one. The rationality of this approach is reflected in three aspects: First, most of the selected independent variables exhibit significant time trends, and the Prophet model can effectively fit their historical changes and extrapolate trends. Second, Prophet has strong capabilities in nonlinear fitting and handling sudden changes, making it suitable for medium- and long-term predictions of socio-economic variables. Finally, by decoupling time series prediction from regression modeling, we have enhanced the model's structural clarity and interpretability

Table 5.Predicted values of socially relevant variables for aging from 2025 to 2034

Time (Year)	Per capita consumption expenditure of residents (RMB)	Per capita consumption expenditure of the elderly (RMB)	Per capita health care expenditure for the elderly (RMB)	Number of pension service institutions (10,000)	...	Total population (10,000 people)
2025	30418.18	34868.83	12263.57	23.37	...	141461.88
2026	31744.58	36188.78	12855.27	23.72	...	141719.12
2027	33066.77	37451.75	13430.53	23.91	...	142027.77
2028	34440.10	39681.05	13394.82	27.07	...	141398.70
2029	35770.22	41058.37	14002.81	27.59	...	141601.72
2030	37096.62	42378.32	14594.51	27.94	...	141858.96
2031	38418.81	43641.29	15169.77	28.12	...	142167.60
2032	39792.14	45870.59	15134.06	31.29	...	141538.54
2033	41122.26	47247.91	15742.05	31.81	...	141741.55
2034	42448.66	48567.86	16333.75	32.16	...	141998.79

As shown in Figure 30, the fitting and forecasting curves of China's GDP from 1995 to 2034 reveal several significant economic phenomena and structural trends. Overall, the trend of economic growth is stable and positive over the long term. The ElasticNet regression model successfully captures the historical GDP growth trend and continues this trend for the next decade, indicating that China's economic growth has intrinsic resilience and potential for sustainable development, even as the aging population intensifies . However, the future growth rate is expected to slow down, reflecting the transition of China's economy from high-speed growth to high-quality growth . This also highlights the pressure on economic growth from structural issues such as population aging and diminishing returns on capital. Further analysis shows that the positive contribution of the silver economy variables to GDP has been incorporated into the model, suggesting that an aging society does not necessarily hinder economic growth but may instead generate new demand, creating a' silver dividend' .

Figure 18.Comparison of predicted GDP and actual GDP of the ElasticNet model



4.4 Pension System Modeling and Simulation Analysis

As China enters a stage of profound population aging, the imbalance between pension revenue and expenditure has emerged as a critical concern for long-term macroeconomic stability and social security sustainability. In particular, with the rapid increase in the elderly population and the relative decline in the working-age population, the traditional pay-as-you-go pension scheme is under dual pressure—weak income growth and rigid expenditure escalation. To quantitatively characterize this structural disequilibrium process and provide a scientific basis for pension reform and fiscal risk forecasting, this study introduces a coupled differential equation system. From a dynamic system modeling perspective, it treats pension income and expenditure as interrelated processes evolving over time, calibrated with empirical data and simulated for future trends.

This method captures the dynamic interactions between key variables within the pension system and provides predictive capacity for future financial gaps, offering both explanatory and forecasting value for policy-making.

4.4.1 Coupled Pension System Model

Given the long-term interplay between the growth of the elderly population, changes in consumption patterns, and institutional factors affecting pension expenditures and revenues in our country, a dynamic system model based on coupled differential equations was chosen to depict the evolutionary paths and lag effects among variables. This model is selected to dynamically simulate the nonlinear trends in pension income and expenditure under the backdrop of an aging population, providing mathematical support for medium-and long-term policy evaluations. To achieve this, five variables were selected: the proportion of the elderly population *A(t)*, per capita consumption by the elderly *C(t)*, per capita medical expenses by the elderly *M(t)*, total pension expenditures *P(t)*, and pension income $P_{in}(t)$. These variables represent three key dimensions: population structure, behavioral spending, and institutional finance, enabling a coupled modeling approach that shifts from ‘aging-driven’ to ‘institutional response.’

Linear regression prediction model of pension income :

$$P_{in}(t) = \theta_0 + \theta_1 t \tag{10}$$

Construction of four-dimensional dynamic coupled differential system:

$$\begin{cases} \frac{dA}{dt} = \alpha(1 - A) \\ \frac{dC}{dt} = \beta A - 0.05C \\ \frac{dM}{dt} = 0.1A + 0.05C - 0.1M \\ \frac{dP}{dt} = \delta A - \gamma P \end{cases} \tag{11}$$

among：

α: Growth rate of the elderly population.

β: The coefficient of the elderly population’s influence on consumption level.

γ: Automatic adjustment (suppression) coefficient of pension expenditure .

δ: The marginal impact intensity of the proportion of elderly population on pension expenditure .

4.4.2 Parameter Estimation and Objective Function

To ensure predictive accuracy and empirical relevance, model parameters are estimated by minimizing the Mean Squared Error (MSE) between the observed and simulated pension income $P_{in}(t)$ and expenditure *P(t)*:

$$\tau(\alpha, \beta, \gamma, \delta) = MSE(P_{real}, P_{pred}) + MSE(P_{in,real}, p_{in,pred}) \tag{10}$$

The scipy.optimize.minimize function is employed to solve this numerical optimization problem under predefined parameter bounds. The estimation results are shown in Table 6.

Table 6.Parameter interpretation of coupled differential equation model

Parameter symbols	estimated value	meaning
α	0.0286	The average annual growth rate of the elderly population is 2.86%
β	30000.00	Every unit of increase leads to 30,000 yuan of consumption growth
γ	0.0010	The ability to adjust expenditure is very weak,and the growth is expanding with inertia
δ	5739.74	Each unit of increase brings an increase in expenditure of 574 billion yuan

4.4.3 Simulation Results and Policy Implications

Using the estimated parameters, we simulate pension system dynamics from 1995 to 2034. Figures 19 and 20 depict the trajectories of expenditure and income, respectively.

Figure 19. Pension Expenditure Trend (1995–2034)

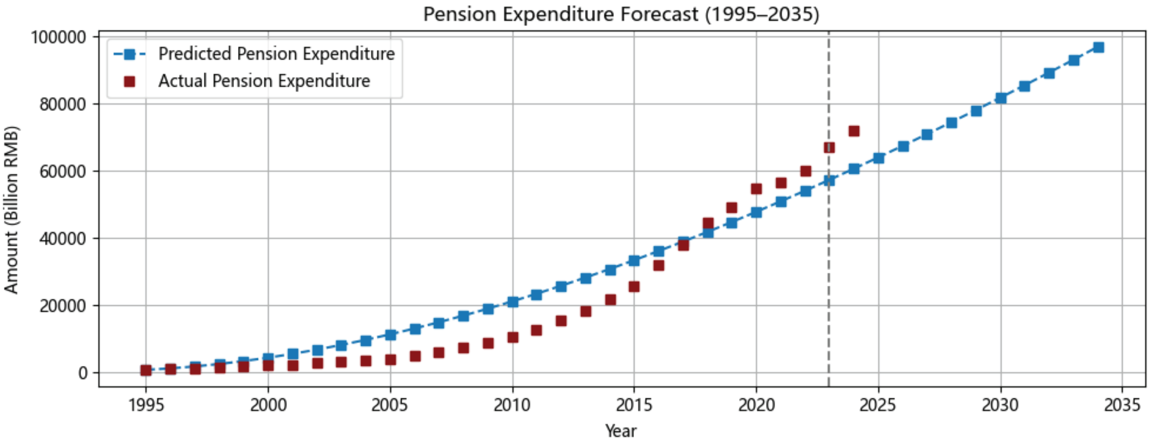
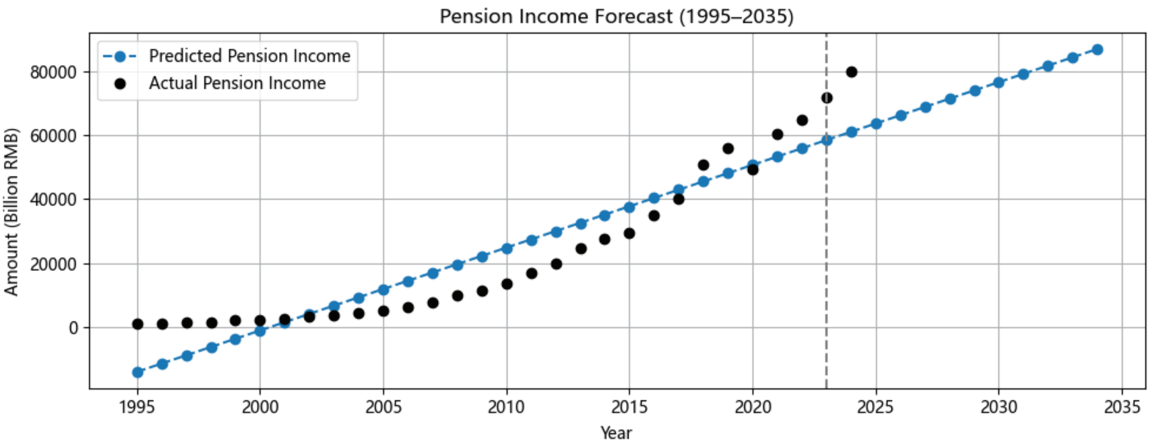


Figure 20. Pension Income Trend (1995–2034)



(1) Expenditure Trends (See Figure 31)

As shown, pension outlays have surged since 2005, driven by rapid population aging, inflexible replacement rates, and rising wage bases. The projection indicates a continued upward trend, signaling mounting fiscal pressure.

(2) Revenue Trends (See Figure 32)

Compared with expenditures, revenue growth is much slower. Although there is a slight rebound after 2015, it remains insufficient to offset the accelerating expenditure trajectory.

(3) Fiscal Risk Forecast

If current institutional parameters remain unchanged, the pension system is expected to face a widening financial gap post-2030. The model predicts structural imbalance, which could strain fiscal sustainability. These findings underscore the urgency of policy reforms such as delayed retirement, adjusted contribution rates, or recalibrated replacement ratios.

5. Policy Recommendations

As China transitions into a deeply aging society characterized by “structural aging” and “getting old before getting rich,” it faces mounting challenges in sustaining economic vitality, social welfare, and demographic balance. Based on comprehensive analyses using LSTM forecasting, ElasticNet regression, and coupled differential modeling, this study proposes a three-tiered policy framework that aims to promote well-being among the elderly, stimulate the silver economy, and ensure the long-term sustainability of the pension system.

5.1 Recommendations Based on LSTM Forecasting: Enhancing Well-Being and Regional Equity

The LSTM-based happiness prediction results reveal that elderly well-being is shaped by multidimensional factors, with mental health, life satisfaction, and physical comfort identified as the most influential determinants. Additionally, spatial disparities in happiness levels across provinces (see Figure 20 and Figure 21) underscore the uneven distribution of public services, infrastructure, and psychological support systems. To address these issues, the following measures are recommended:

5.1.1 Strengthen Public Services and Mental Health Support

Invest in elderly-centered public services—especially in healthcare, eldercare, and community infrastructure—across underserved regions. Mental health services should be integrated into rural and aging urban communities and incorporated into regional performance evaluation systems to incentivize improvements in well-being delivery.

5.1.2 Promote Social Participation and Community Integration

Encourage senior involvement in volunteer services, cultural activities, and neighborhood networks to enhance psychological engagement, mitigate loneliness, and improve overall life satisfaction.

5.1.3Implement Regional Monitoring and Feedback Mechanisms

Establish a real-time monitoring framework for happiness indices to facilitate targeted resource allocation, responsive policy adjustments, and coordinated regional development.

5.2 Recommendations Based on ElasticNet Regression: Stimulating the Silver Economy as a New Growth Driver

ElasticNet regression analysis demonstrates that elderly consumption and service demand have significant positive effects on economic growth. Specifically, elderly participation in economic and social activities and the supply of eldercare services exhibit a strong correlation with GDP growth (see Figures 26–30). These findings support the development of a silver economy as a strategic national priority. Key policy actions include:

5.2.1 Build a High-Quality Silver Economy Industrial Chain

Develop an integrated eldercare industry that includes healthcare, smart assistive technology, senior-friendly housing, medical tourism, and lifelong learning. Leverage fiscal incentives, land-use policies, and financial instruments to mobilize private investment and technological innovation in aging-related sectors.

5.2.1 Stimulate Age-Friendly Consumption and Unlock Domestic Demand

Introduce consumption subsidies, promote senior-exclusive financial products, and pilot "age-friendly consumption zones" to stabilize silver spending expectations and boost structural domestic demand.

5.2.3Encourage Multi-Stakeholder Engagement in Eldercare Services

Strengthen the third pillar of the pension system by incentivizing private pension plans and long-term care insurance. Enable public-private partnerships to expand service coverage, improve care quality, and promote integrated development across different regions.

5.3 Recommendations Based on the Coupled Model: Ensuring Long-Term Pension Sustainability

The coupled differential equation model projects that without policy reform, the pension expenditure gap will widen substantially due to rapid population aging (see Figures 31–32). To ensure fiscal sustainability and intergenerational equity, the following integrated strategies are advised:

5.3.1 Establish a Dynamic Contribution and Retirement Framework

Gradually raise the statutory retirement age in accordance with rising life expectancy and labor supply pressures. Develop a flexible, growth-aligned contribution mechanism that adjusts with changes in demographic and economic conditions.

5.3.2 Expand the Third Pillar and Alleviate Fiscal Pressure

Accelerate the development of commercial pension insurance and personal retirement accounts to diversify income sources for future retirees and reduce dependence on the basic pension scheme.

5.3.3 Optimize Pension Investment Portfolios

Enhance the return on pension fund investments by expanding the share of equity-based and long-duration assets, including real estate investment trusts (REITs), infrastructure, and public utilities, under prudent risk control frameworks.

5.4 Conclusion:

China's aging challenge also presents an opportunity to modernize its economic structure, expand domestic demand, and deepen social reform. Policy interventions rooted in data-driven modeling, regional differentiation, and institutional innovation can help transition toward a "triple-win" scenario of enhanced well-being, resilient growth, and sustainable public finance.

6. Conclusion and Research Outlook

6.1 Main Findings

This study constructs a comprehensive quantitative modeling framework to systematically analyze the macroeconomic implications of population aging and the potential of the silver economy in China. The key findings are summarized as follows:

6.1.1 The aging trend is accelerating, posing structural challenges

Based on population data from 1995 to 2024, the LSTM model reveals a persistent upward trend in the proportion of the population aged 65 and above, expected to exceed 20% by 2035. This trajectory is driven by declining birth rates, increasing life expectancy, and delayed generational replacement, exhibiting strong nonlinearity and calling for early policy responses.

6.1.2 Elderly well-being is shaped by multidimensional factors with regional disparities

Utilizing Bayesian regression on CHARLS micro-survey data, we identify psychological well-being, life satisfaction, and physical health as the most influential drivers of elderly happiness, with 95% HDIs excluding zero. Significant regional heterogeneity in well-being levels suggests the need for differentiated interventions in mental health, infrastructure provision, and community-based services.

6.1.3 The silver economy holds growth potential, but structural mismatches remain

From a macro perspective, ElasticNet regression demonstrates that elderly consumption power is positively correlated with GDP, and the adequacy of service supply is a critical determinant of macroeconomic performance. However, China's silver economy is still in a formative phase, facing supply-demand mismatches in health care, age-friendly technologies, and elder services, which constrain its full potential.

6.1.4 Pension fund sustainability is at risk, requiring dynamic regulation mechanisms

A coupled differential equation model simulates pension fund revenues and expenditures from 1995 to 2035. The results show an accelerating expenditure trend surpassing income growth, indicating long-term fiscal unsustainability. Without institutional reforms in contribution rates and investment mechanisms, the pension system may exert fiscal pressure and threaten intergenerational equity.

6.2 Future Research Directions

Despite the development of a comprehensive modeling system, several areas warrant further exploration:

6.2.1 Dynamic model updating and cross-country comparison

Current forecasting models rely primarily on historical data. Future research could incorporate real-time data sources (e.g., social media, digital medical records) for adaptive updating. Moreover, building comparative models for Japan, South Korea, and European economies may offer international benchmarks and contextualize China's aging challenges.

6.2.2 Integration of behavioral economics and policy feedback mechanisms

This study adopts regression-based and differential modeling approaches, which do not fully capture policy behavior feedback. Agent-Based Models (ABM) and system dynamics could be introduced to simulate elderly responses to policy reforms (e.g., pension adjustments, medical subsidies), thereby enhancing scenario analysis and long-term impact evaluation.

6.2.4 Fiscal sustainability and policy scenario assessment

Further research may focus on simulating the fiscal consequences of alternative policy proposals, such as delayed retirement, pension account reform, and expansion of occupational pensions. This could enable quantitative evaluation of reform trade-offs and support data-driven institutional optimization.

In conclusion, population aging presents both profound challenges and transformative opportunities for China's economic and social development. By constructing a unified system that links trend prediction, well-being assessment, economic contribution, and institutional response, this study lays a foundation for interdisciplinary policy modeling and forward-looking strategic planning.